Honours Topics

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Any pure research or applied topic of your interest in Probability theory, Stochastic/Statistical Modelling, Markov Chains, Simulation, or other areas of Operations Research. If you are interested in analysis involving uncertainty, or constructing models for unpredictable real-life systems, or inventing algorithms and coding them, I can find an interesting project for you. Please see the examples below.

Phase-Type Distributions and their Applications

(currently available)

Summary: First, recall the exponential distribution definition. We say that a random variable X follows *exponential* distribution with (rate) parameter $\lambda > 0$, and write $X \sim \text{Exp}(\lambda)$, when its density function $f(t)$ is

$$f(t) = \begin{cases} \lambda e^{\lambda t} & \text{when } t \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Phase-Type (PH) distribution is a generalization of the exponential distribution, which is defined as follows. Let $\{J(t)\}$ be an *absorbing* continuous-time Markov Chain with state space $\mathcal{S} = \{0, 1, \ldots, m\}$, where 0 denotes an absorbing state, and rates matrix $\mathbf{Q} = [q_{ij}]$ partitioned as

$$\mathbf{Q} = \begin{bmatrix} 0 & \mathbf{0} \\ \underline{\mathbf{t}} & \mathbf{T} \end{bmatrix},$$

where the (square) matrix $\mathbf{T} = [q_{ij}]_{i,j \in \mathcal{S} \setminus \{0\}}$ records the transition rates between the nonabsorbing states, and the (column) vector $\underline{\mathbf{t}} = [q_{i,0}]_{i \in \mathcal{S} \setminus \{0\}}$ records the transition rates from nonabsorbing states to the absorbing state 0. Also, let $\underline{\boldsymbol{\alpha}}$ be the initial distribution (row) vector $\underline{\boldsymbol{\alpha}} = [\alpha_i]_{i \in \mathcal{S}}$.

We say that a random variable X follows **Phase-Type (PH)** distribution with parameters $\underline{\boldsymbol{\alpha}}$, $\mathbf{T}$, and $\underline{\mathbf{t}}$, and write $X \sim \text{PH}(\underline{\boldsymbol{\alpha}}, \mathbf{T}, \mathbf{t})$, when its density function $f(t)$ is given by

$$f(t) = \begin{cases} \underline{\boldsymbol{\alpha}} e^{\mathbf{T}t} \underline{\mathbf{t}} & \text{when } t \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

The cumulative distribution function is then given by

$$F(t) = P(X \leq t) = \begin{cases} 1 - \underline{\boldsymbol{\alpha}} e^{\mathbf{T}t} \underline{\mathbf{1}} & \text{when } t \geq 0 \\ 0 & \text{otherwise,} \end{cases}$$

where $\underline{\mathbf{1}}$ denotes a column vector of ones of appropriate size.

As an example of application, if X records the duration of *lifetime* (eg of a gene), then we interpret $f(t)$ as the density of the lifetime ending at the precise time point t, $F(t)$ as the probability of it ending before time t, and $(1 - F(t))$ as the probability of surviving at least t units of time.

PH distribution has various interesting applications and connections to a wide range of models and problems. As example, it can be used to approximate any positive-valued distribution. The beauty of the PH distribution is that various complex problems (eg general sum of exponentially distributed random variables) can be solved in a simple and elegant way. This project will focus on the review of the literature in the theory of PH distribution and its applications, and will include some numerical work.

Reference:

[1] G. Latouche, V. Ramaswami. Introduction to Matrix Analytic Methods in Stochastic Modelling, 1st edition. Chapter 2: PH Distributions; ASA SIAM, 1999.

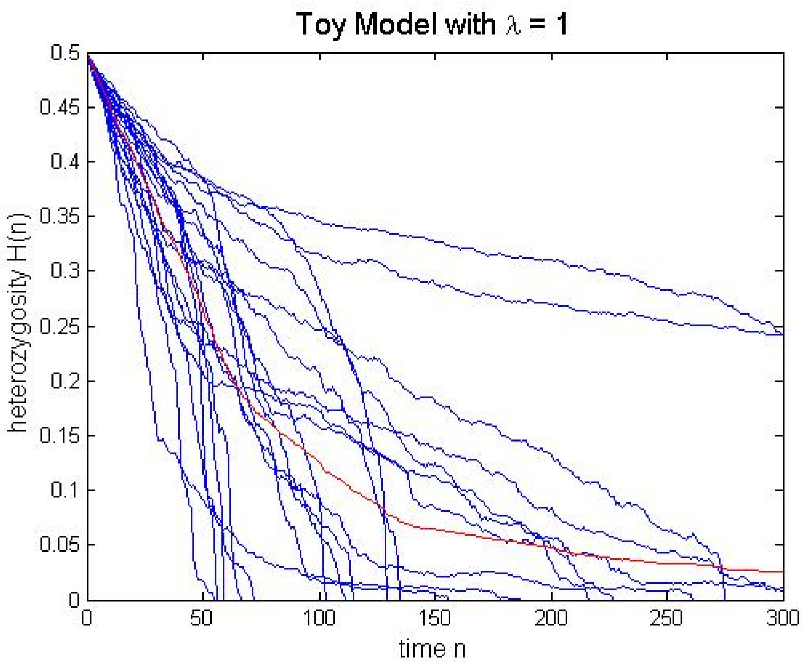
Stochastic Models for the Conservation of Endangered Species.

(currently available)

Summary: In populations of endangered species, management strategies referred to as *genetic rescue* have been advocated in order to help avoid extinction. An example of considerable concern in the Australian context is the conservation management of Tasmanian Devils suffering from the *Devil Facial Tumour Disease* (DFTD), which puts them in danger of extinction. An important factor in this context is the ability to assess the impact of conservation efforts. Conservation strategies have been used with the hope of increasing the genetic diversity of the wild population, but this remains a challenging problem.

This project will focus on models for the numerical assessment of conservation strategies, which will assist in these efforts. This project will involve reviewing the literature in the area and numerical experimentation using simulation models.

You will have an opportunity to be part of a rich collaborative environment and interact with mathematicians and biologists studying the wild populations of Tasmanian Devils.



Selected examples of past topics:

Stochastic Fluid Models and their Applications

(Mr Adrian Tanner, Honours, work in progress)

Summary: Stochastic Fluid Models (SFMs) are a class of models with a two-dimensional state-space $\{\varphi(t), X(t)\}$ consisting of a phase $\varphi(t) \in \mathcal{S}$ and a level $X(t) \in \mathbb{R}$. The phase variable $\varphi(t)$ is often used to describe the state of some physical environment that we wish to model. Simple two-phase examples are on/off mode of a switch in a telecommunications buffer, peak/off-peak period in a telephone network, or wet/dry season in reservoir modeling. The level variable $X(t)$ is used to model some continuous performance measure of the system, eg the amount of data in a buffer, the level of water in a reservoir, or the revenue earned by time t. The model assumes that the transitions between phases occur according to some underlying continuous-time Markov Chain (CTMC) with some state space $\mathcal{S}$ and rates matrix $\mathbf{T} = [T_{ij}]$, with $T_{ij} = dP(X(t) = j|X(0) = i)/dt$. Furthermore, the rate $c_i = dX(t)/dt$ of increase of the fluid level at time t depends on the phase $\varphi(t) = i$ at time t, and so the Markov Chain is the process that *drives* the fluid level at time t.

The aim of the project is to review the literature in the area of applications of SFMs and construct numerical examples using efficient algorithms and simulations. This project will suit a student who is interested in stochastic modeling, analysis and algorithmic approaches (and coding).

Queueing Models for Bed Queue in Emergency Department

(Mr Jarrad Clark, Honours, 2015/2016)

Summary: Modern hospital is a highly complex and unpredictable system, which cannot be managed efficiently using intuitive methods. Instead, we require sophisticated tools in the form of efficient algorithms developed using appropriate mathematical modeling. Compelling clinical evidence indicates that when mathematical modeling is used in hospitals, significant savings can be made that have a positive outcome to the patients.

The aim of this project was to review the literature in the area of modelling the patients flow in Bed Queue in Emergency Department. The project also involved construction of models and the derivation of various performance measures based on the analysis of a large set of data.

Markov Models for Microsatellite Mutation

(Mr Tristan Stark, Honours, 2013)

Summary: Markov Chains is the most important class of models in Probability Theory, due to their modeling potential and numerical tractability. The aim of this project was to review the literature concerning the application of Markov Chains in Phylogenetics, the study of evolutionary relation among groups of organisms (e.g. species, populations), for example see:

Calabrese, P., Sainudiin, R., Models of Microsatellite Evolution, in Statistical Methods in Molecular Evolution, R. Nielsen, Editor. 2005, Springer, p. 289-306.

Stochastic Fluid Model for Deteriorating Systems

(Mr Andrew Haigh, Honours, 2012)

Summary: Markovian-modulated models are a class of models with a two-dimensional state-space consisting of a phase and a level. The phase variable is often used to describe the state of some physical environment that we want to model. Simple two-phase examples are on/off mode of a switch in a telecommunications buffer, peak/off-peak period in a telephone network, or wet/dry season in reservoir modeling. The model assumes that the transitions between phases occur according to some underlying continuous-time Markov Chain. Furthermore, the rate of increase of the fluid level at time t depends on the phase at time t, and so the Markov Chain is the process that drives the fluid level at time t. The aim of the project was to explore and review the current literature in the area.